TEMPERATURE FIELD OF AN INFINITE PLATE WITH VARIABLE THERMOPHYSICAL CHARACTERISTICS

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Nomograms are proposed to compute the temperature fields of simplest kind with a linear change in the thermophysical coefficients for boundary conditions of the third kind, which refine engineering computations of body heating.

Let us approximate the temperature dependence of the specific heat C, J/m^3·deg and of the coefficient of heat conduction λ, W/m·deg by the linear expressions

$$\lambda = \lambda_0 (1 + m\vartheta); \quad C = C_0 (1 + n\vartheta), \tag{1}$$

where $\vartheta = (t - t_0)/(t_c - t_0)$; t_c is the temperature of the medium; t_0 is the initial body temperature. The temperature field of a plate $\vartheta(X, Fo)$ is described by the equation

$$(1 + n\vartheta) \frac{\partial \vartheta}{\partial Fo} = \frac{\partial}{\partial X} (1 + m\vartheta) \frac{\partial \vartheta}{\partial X} \tag{2}$$

TABLE 1

Bi	Fo	1		2		3
		$\vartheta_1(1, Fo)$	δ_{S_1}, %	$\vartheta_2(1, Fo)$	δ_{S_2}, %	$\vartheta_3(1, Fo)$
0,2	0,02	51,16	1,600	50,48	2,910	51,99
	0,04	63,60	−0,267	63,07	0,567	63,43
	0,06	72,98	−0,223	72,51	0,425	72,81
	0,1	87,56	−0,183	87,16	0,275	87,40
1,0	0,005	95,09	−1,331	90,13	3,954	93,84
	0,01	123,57	−0,595	120,00	2,314	122,84
	0,05	229,63	−0,178	228,00	0,532	229,22
	0,1	296,42	−0,142	295,28	0,244	296,00

TABLE 2

	$\theta = 1 - \vartheta_s = 0,1$											
	Bi=5				Bi=1				Bi=0,5			
	δ_S, %	δ_S, %	δ_S^*, %	δ_C^*, %	δ_S, %	δ_C, %	δ_S^*, %	δ_C^*, %	δ_S, %	δ_C, %	δ_S^*, %	δ_C^*, %
$m=2$ $n=1$	7,3	−2,64	1,3	1,34	10,95	−1,57	1,17	0,46	14,2	6,2	0,77	0,33
$m=-0,7$ $n=-0,7$	−30	28,8	2,5	3,03	−17,3	25,9	3,06	1,96	−18,2	17,5	2,36	1,2

	$\theta = 1 - \vartheta_s = 0,05$											
	Bi=5				Bi=1				Bi=0,5			
	δ_S, %	δ_C, %	δ_S^*, %	δ_C^*, %	δ_S, %	δ_C, %	δ_S^*, %	δ_C^*, %	δ_S, %	δ_C, %	δ_S^*, %	δ_C^*, %
$m=2$ $n=1$	7,7	−20	1,99	0,77	10,1	−11,0	2,68	0,9	12,6	−9,3	−9,6	0,21
$m=-0,7$ $n=-0,7$	−25	38,4	0,46	5,7	−28,0	38,8	4,1	1,56	−27,7	27,8	1,8	0,55

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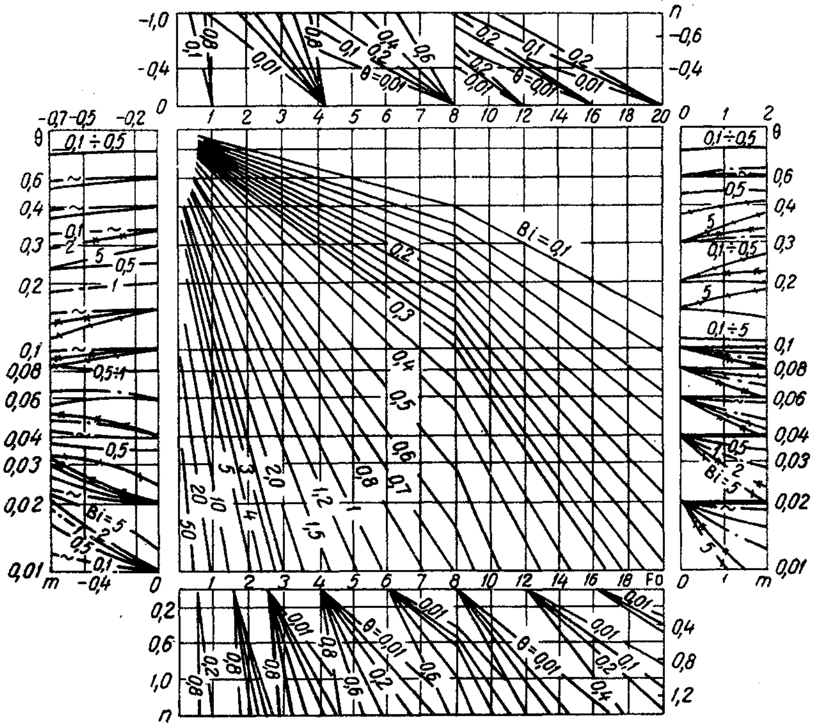

Fig. 1. Time to heat the surface of an infinite plate ($\theta = 1 - \vartheta_S$) for $t(x, 0) = t_0$ in a medium with t_C = const if the thermophysical coefficients of the body are given by the equations $\lambda = \lambda_0 (1 + m\vartheta)$ and $c = c_0 (1 + n\vartheta)$, where $\vartheta = (t - t_0)/(t_C - t_0)$.

with the boundary and initial conditions

$$(1 + m\vartheta) \frac{\partial \vartheta}{\partial X} (1,\ \mathrm{Fo}) = \mathrm{Bi}\,[1 - \vartheta(1,\ \mathrm{Fo})],$$

$$\frac{\partial \vartheta}{\partial X} (0,\ \mathrm{Fo}) = 0; \quad \vartheta(x,\ 0) = 0, \tag{3}$$

where

$$\mathrm{Fo} = \frac{\lambda_0 \tau}{C_0 R^2}; \quad X = \frac{x}{R}; \quad \mathrm{Bi} = \frac{\alpha R}{\lambda_0}. \tag{4}$$

The solution has been obtained on a "Ural-2" electronic computer by finite differences [1]. Checking computations of the heating of a plate with constant coefficients were performed to estimate the error in the solution for a discrete representation of the temperature field. Presented in Table 1 is the temperature ϑ (1, Fo) of the plane being heated as found by our solution (column 1), by the solution in [2] (column 2), and by the exact solution [3] (column 3), and the corresponding relative errors are given in percents.

We have worked out the computed temperature field of a plate with variable coefficients in the form of nomograms Fo = f($\theta = 1 - \vartheta$, Bi, m, n, x = const), which are awkward and inconvenient for engineering computations in the general case. Simplified nomograms (Figs. 1 and 2) are proposed for engineering computations. The central field of the nomograms duplicates the known D. V. Budrin nomogram

$$\theta = 1 - \vartheta = f\left(\mathrm{Fo} = \frac{\lambda_0 \tau}{C_0 R^2}; \quad \mathrm{Bi} = \frac{\alpha R}{\lambda_0}; \quad m = 0; n = 0; \quad x = \mathrm{const}\right)$$

Corrections for the variability of the thermophysical coefficients of the body reduce to the central section on the fields.

The horizontal lines ϑ = const of the D. V. Burdin nomogram branch out in the fields into families of curves in values of the Biot criterion, and the vertical lines Fo = const in families of values of the criterion θ.

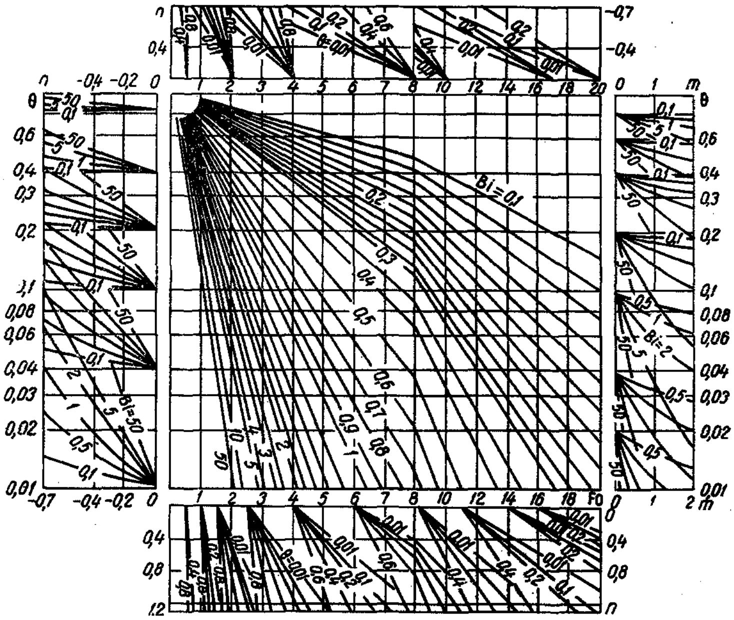

Fig. 2. Time to heat the middle surface of a plate ($\theta = 1 - \vartheta_C$). Conditions and thermophysical characteristics the same as in Fig. 1.

If $m = m_1$ and $n = 0$ are given in (1), then for the values θ_1 and Bi_1 of interest to us the determination of Fo_1 starts with establishing the intersection of the vertical m_1 = const with the horizontal θ_1 = const. We emerge from the intersection to the boundary of the central field of the nomogram along the line $Bi = Bi_1$ at some provisional value θ_*. Upon intersecting the curve $Bi = Bi_1$ on the D. V. Budrin nomogram the horizontal θ_* = const determines the value Fo_1 directly on this nomogram.

If $m = 0$ and $n = n_1$ are given in (1), then the auxiliary value Fo_* corresponding to the case $m = 0$, $n = 0$ is first found on the D. V. Budrin nomogram. The correction for $m \neq 0$ is achieved by finding the intersection of the curve ($Fo = Fo_*$, $\theta = \theta_1$) with the line $n = n_1$.

If $m = m_1$ and $n = n_1$ are given, then an auxiliary value Fo_1 corresponding to the case $n = 0$ is determined first, and the correction for $n \neq 0$ which yields the final desired value of Fo is found from the condition $Fo_* = Fo_1$.

As an illustration of the use of the nomograms, let us determine the time to heat a 1Kh18N9V steel plate of nominal thickness $R = 0.025$ m in a medium with $t_c = 1200$°C = const and $\alpha = 310$ kJ/m · °C from the initial temperature $t_0 = 0$°C to $t(R, \tau_k) = 800$°C.

The thermophysical properties of 1Kh18N9V steel are approximated linearly in the 0-800°C temperature range: $\lambda(\vartheta) = 15.1\,(1 + 1.87\vartheta)$ W/m · °C, $C(\vartheta) = 0.537\,(1 + 0.54\vartheta)$ kJ/kg · °C.

For the computation by the nomograms we have $Bi = \alpha R/\lambda_0 = 0.513$; $m = 1.87$; $n = 0.54$; $\theta_0 = 1 - \vartheta_S = 0$.

We find $Fo_1 = 1.8$ (Fig. 1) for $Bi = 0.513$; $\theta = 1 - \vartheta_S = 0.4$ and $m = 1.87$. The correction for the value $n = 0.54$ yields the final answer $Fo = 2.13$ or $\tau = (2.13 \cdot 0.025^2/3.61 \cdot 10^{-6})\,3600 = 0.106$ h.

For average thermophysical coefficients ($\bar{a} = 4.72 \cdot 10^{-6}$ m/sec, $\bar{\lambda} = 20.9$ W/m · deg) we have $\tau = 0.0864$ h by the D. V. Budrin nomograms.

Presented in Table 2 are the error δ, % of the temperature computation of the center of the body, and the time of its heating for a given temperature of a surface with variable coefficients according to the D. V. Budrin nomograms for average and constant coefficients and by the simplified nomograms (Figs. 1 and 2).

As can be seen from Table 2, the error in using the simplified working nomograms δ^*, % is an order of magnitude less than the error δ, % of a standard engineering analysis for constant average coefficients.

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